[3]Kerschen, E. J. and Balsa, T. F., "Transformation of the Equations Governing Disturbances of Two-Dimensional Compressible Flow," *AIAA Journal*, Vol. 19, Oct. 1981, pp. 1367-1370.

[4]Goldstein, M. E., "Scattering and Distortion of the Unsteady Motion on Transversely Sheared Mean Flows," *Journal of Fluid Mechanics*, Vol. 91, 1979, pp. 601-632.

[5]Lin, C. C., *The Theory of Hydrodynamic Stability*, Cambridge University Press, London, 1966.

[6]Lilley, G. M., "On the Noise from Jets," AGARD CP-131, 1974.

Transient Decay Times and Mean Values of Unsteady Oscillations in Transonic Flow

Earl H. Dowell* and Tetsuhiko Ueda†
Princeton University, Princeton, New Jersey
and
Peter M. Goorjian‡
NASA Ames Research Center
Moffett Field, California

Introduction

IN Ref. 1, Kerlick and Nixon have made the important point that, when using a finite difference, time-marching computer code to investigate the lift on an oscillating airfoil in transonic flow, it is necessary to carry the solution sufficiently far forward in time that an essentially steady-state solution is obtained. Moreover, they offer a method for estimating the transient time before the steady state is reached. They note that, if one stops the time-marching solution before the transient is complete and the steady state is reached, then one may reach the incorrect conclusion that a change in the mean lift has occurred due to the oscillating motion of the airfoil when in fact no such change has occurred.

Nevertheless, what is perhaps surprising, and the principal reason for the present study, is that for a narrow Mach number range the time for the transient to decay and a steady state to be reached is extraordinarily long. Moreover, for a very narrow range of Mach number a nonzero mean value of lift can occur for an airfoil of symmetrical profile oscillating about a zero angle of attack.

Technical Discussion

In Ref. 1 a NACA 64A006 airfoil was studied using the LTRAN2 computer code for a freestream Mach number of 0.875 with the airfoil oscillating at a peak angle of attack of 0.25 deg and with various reduced frequencies k. It was shown that typically up to six cycles of airfoil oscillation must be considered for the mean lift to be less than 1% of the corresponding oscillatory lift peak value. In Ref. 2 the first author and his colleagues (also using the LTRAN2 computer code) examined various airfoils (64A006, 64A010, MBB-A3) at various Mach numbers and reduced frequencies. The calculations in Ref. 2 were carried forward in time through six cycles of airfoil oscillation. As was found by Kerlick and Nixon, the results of Ref. 2 for the (symmetric) 64A006 airfoil showed the mean or average lift with the airfoil oscillating to be essentially unchanged from its value for no airfoil oscillation (i.e., zero) for *most* Mach numbers studied *after six cycles of oscillation.* However at $M_\infty = 0.88$ and particularly at $M_\infty = 0.9$ this was *not* the case. Hence, the first author *incorrectly* concluded that a change in mean or average lift had occurred. The correct conclusion is that at $M_\infty = 0.9$ and 0.88 many cycles of oscillation (>40) are required for the mean lift to decay to essentially zero.

The results presented in Fig. 1 are for $M_\infty = 0.9$, $k = 0.2$, and various peak oscillatory angles of attack.[2] They are for the lift coefficient; similar results (not shown for brevity) were obtained for moment coefficient (about the midchord) and shock displacement (normalized by the airfoil chord). Both mean steady values and first harmonic values are shown *after six cycles of oscillation.* As can be seen, the change in mean lift at this Mach number can be as much as 50% of the peak oscillatory lift. For $M = 0.88$ the mean or average components are never more than 10% of the first harmonic oscillatory components and hence these results are not displayed.

Inspired by the Note of Kerlick and Nixon, the case of $k = 0.2$ and a peak oscillating angle of attack of 0.5 deg was

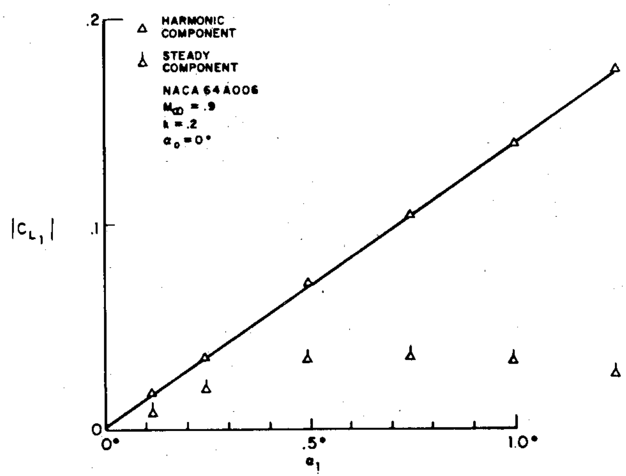

Fig. 1 Steady-state and first harmonic lift components vs dynamic angle of attack, NACA 64A006, $M_\infty = 0.9$, $k = 0.2$, $\alpha_0 = 0$ deg.

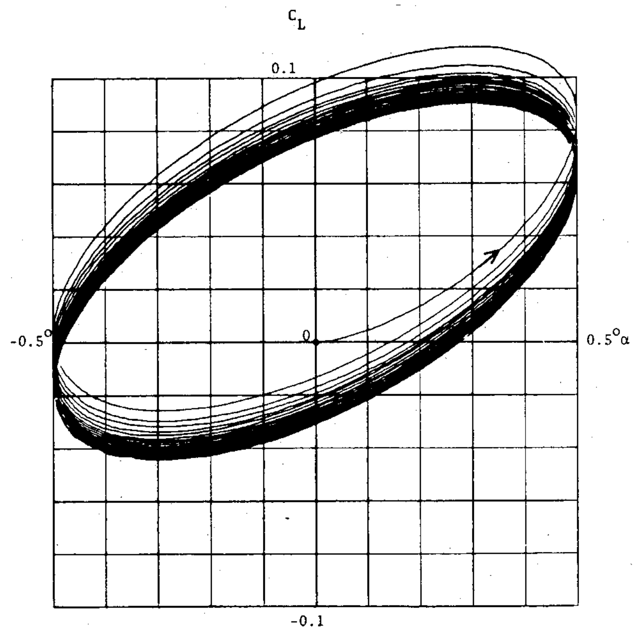

Fig. 2 NACA 64A006 airfoil, $M_\infty = 0.9$, $k = 0.2$.

Received June 11, 1982; revision submitted March 21, 1983.

*Professor, Department of Mechanical and Aerospace Engineering.

†Visiting Fellow, Department of Mechanical and Aerospace Engineering.

‡Research Scientist, Applied Computational Aerodynamics Branch.

carried forward in time through many more cycles of airfoil oscillation for several M_∞ by the first two authors. A typical result is shown in Fig. 2 for $M_\infty = 0.9$ in terms of lift vs angle of attack through 20 cycles of oscillation. Time varies along the (hysteresis) curve. As may be seen after 20 cycles the average lift is still distinctly different from zero and a substantial fraction of the oscillatory lift. The average lift is defined as

$$C_{L_{\text{AVG}}} = (C_L^+ + C_L^-)/2$$

and the oscillatory lift as

$$C_{L_{\text{OSC}}} = (C_L^+ - C_L^-)/2$$

where C_L^+ and C_L^- are peak values, positive and negative, respectively, of any two adjacent peaks.

The essential results can be summarized compactly in Fig. 3 where $C_{L_{\text{AVG}}}$ is plotted vs $1/N$, where N is the cycle number of the angle-of-attack oscillation. The results are plotted vs $1/N$ so that the limit as $N \to \infty$ or $1/N \to 0$ may be more readily examined. As may be seen for $M_\infty = 0.86$ and 0.875, the average lift rapidly declines and is essentially zero for $N > 10$ or $1/N < 0.1$. (However, it should be noted that the oscillatory lift $C_{L_{\text{OSC}}}$ converges much more rapidly than the average lift. For the sake of brevity, $C_{L_{\text{OSC}}}$ is not shown.) By contrast for $M_\infty = 0.9$ and 0.885 the average lift persists in measurable values through a much larger number of cycles. For example, for $M_\infty = 0.9$ at $N = 40$ or $1/N = 0.025$, there is a clear trend toward zero average lift as $N \to \infty$ or $1/N \to 0$, but at $N = 6$ this would be difficult to perceive. Of course, these results have important practical consequences. A run for $N = 40$ takes 1672 s of CPU time on an IBM 3033.

Hence at this point the first two authors concluded that while a nonzero average lift is mathematically possible for a nonlinear aerodynamic system responding to an oscillating angle of attack, no such lift was observed using LTRAN2 for the range of parameters studied. However, at some Mach numbers the time for the average lift to decay to zero is extraordinarily long.

Next the first two authors greatly benefited from a discussion with the third author.[3] He had carried out calculations[4] at $M_\infty = 0.89$ which indicated that a nonzero average lift did occur. Goorjian's results are more fully discussed in the Appendix. Hence, the first two authors also carried out calculations at this M_∞ and the results are shown in Fig. 4 (analogous to Fig. 2) and Fig. 5 (analogous to Fig. 3). These results clearly suggest that a nonzero average lift *does* occur at $M_\infty = 0.89$.

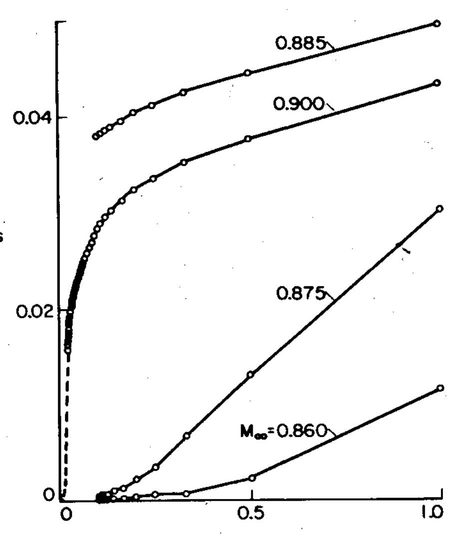

Fig. 3 Average lift vs inverse of the number of cycles of oscillations.

Questions still remain, of course:

1) Why does this nonzero average lift occur only over a narrow range of Mach number?

2) Is it an artifact of LTRAN2 or will other transonic codes exhibit similar behavior?

3) Is the result physically significant? In particular, what would be the counterpart, if any, for a viscous fluid model?

Appendix

Peter M. Goorjian

During the initial calculations with the NASA Ames computer code LTRAN2, a series of calculations was made on a NACA 64A006 airfoil in oscillatory plunging motion at a reduced frequency $k = \omega c/U_\infty = 0.1$. The motion is described by an unsteady angle of attack given by $\alpha = \alpha_I \sin\omega t$. Three values of α_I were used with $\alpha_I = 0.5$, 1.0, and 1.5 deg. The plan was to calculate unsteady loads for freestream Mach numbers M_∞ between 0.80 and 0.99 with increments of 0.01 in Mach number. However I discovered, as described in Ref. 4, that the unsteady loads became periodic about nonzero means values at $M_\infty = 0.89$ and 0.90 for $\alpha_I = 0.5$ deg. Hence the calculations were stopped at $M_\infty = 0.89$. (Figure 5 of this Note shows that in the case $M_\infty = 0.90$, the nonzero mean value is slowly decaying to zero.)

For all of the other cases, the loads became periodic about zero mean values after seven cycles of oscillation, although for the $\alpha_I = 1.5$ deg cases only four cycles of computation were necessary. In the case $M_\infty = 0.89$, $\alpha_I = 0.5$ deg a calculation with $\alpha_I = -0.5$ deg was done and the mean lift also became negative. Finally, 40 cycles of oscillation were calculated to determine if the nonzero mean loads were slowly decaying transients, but it was found that they did not decay. Note that this anomalous behavior occurs for the small-amplitude case, $\alpha_I = 0.5$ deg, but does not occur when α_I is increased to 1.0 and 1.5 deg.

Further calculations have now been made to determine the Mach number range of this nonunique behavior of the flow response to the airfoil motion. The response is nonunique in the sense that two responses can be found for the same periodic motion, where each response is determined by the manner in which the motion is initialized. Calculations were done for $k = 0.1$, $M_\infty = 0.89$-0.99, and $\alpha_I = 0.5$ and 1.0 deg with six cycles of oscillation. The flow response became symmetric in all cases except for $\alpha_I = 0.5$ deg and $M_\infty = 0.89$ or 0.90. This symmetry was displayed in zero mean values for the unsteady loads, as well as unsteady pressure distribution on the airfoil's upper and lower surfaces that became identical after a shift in phase by a half-cycle for one surface.

For all Mach numbers from 0.8 to 0.99, the unsteady flows were initiated from the steady flow for that Mach number. For $M_\infty = 0.80$-0.82 the initial flow is subsonic; for $M_\infty = 0.83$-0.91 the initial flow has normal shocks only, which are located upstream of the trailing edge; and for $M_\infty = 0.92$-0.99 the initial flow has a fishtail shock wave pattern with oblique shocks at the trailing edge and a normal shock downstream of the airfoil.

Next a calculation, using 12 cycles of oscillation, was performed to determine if the nonzero mean loads in the case with $M_\infty = 0.89$, $\alpha_I = 0.5$ deg, and $k = 0.1$ would remain if the airfoil motion was stopped. For cycles 1-3,

$$\alpha = \alpha_I \sin kt$$

For cycles 4-6,

$$\alpha = \alpha_I \sin kt\,(0.5)\,\{1 + \cos[(kt/6) - \pi]\}$$

Finally $\alpha = 0$ for cycles 7-12. The lift response for the first three cycles was similar to that shown in Fig. 4a, where $k = 0.2$. During cycles 4-6, the lift coefficient was in damped oscillation toward $C_L \cong 0.181$. From the end of the sixth cycle to the end of the twelfth cycle, the C_L value grew

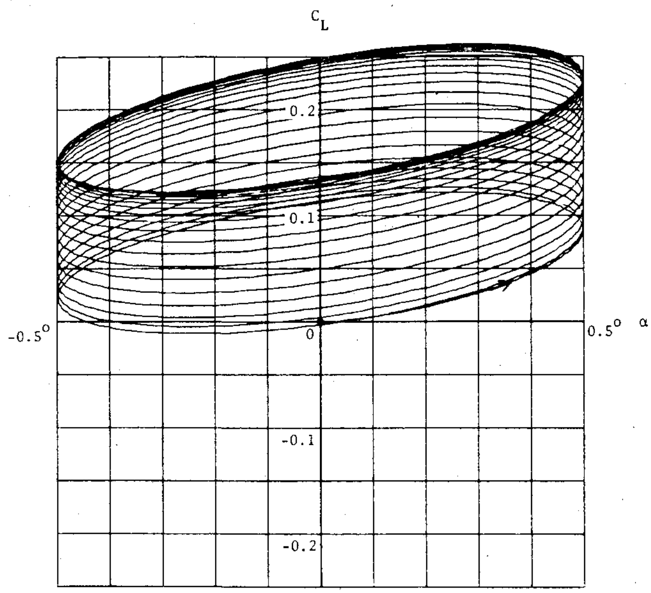

Fig. 4a NACA 64A006 airfoil, $M_\infty = 0.89$, $k = 0.2$, $N = 1 \sim 20$ cycles.

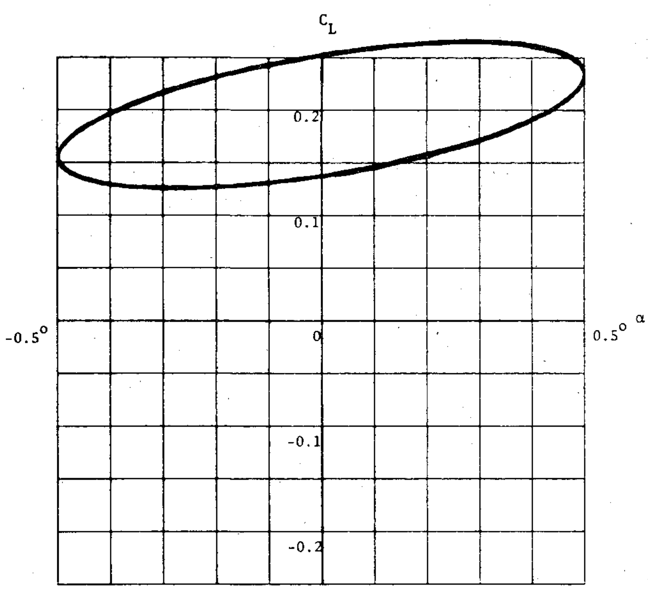

Fig. 4b NACA 64A006 airfoil, $M_\infty = 0.89$, $k = 0.2$, $N = 21 = 40$ cycles.

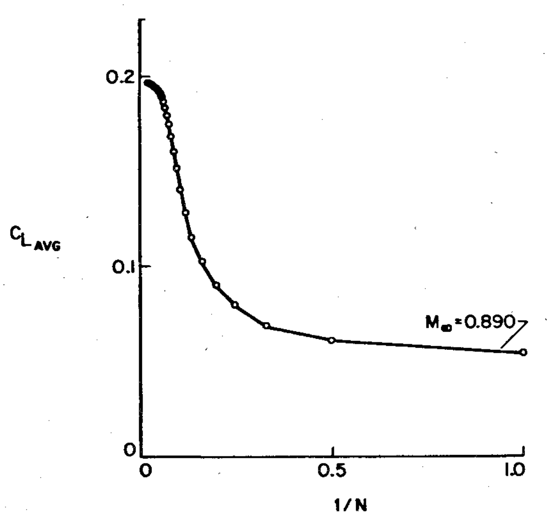

Fig. 5 Average lift vs inverse of the number of cycles of oscillations.

monotonically. The values of C_L at the end of cycles 6-12 were 0.1867 (6), 0.2010 (7), 0.2072 (8), 0.2118 (9), 0.2148 (10), 0.2169 (11), and 0.2181 (12). Hence, the conclusion is that at these flow conditions the symmetric steady flow is unstable and that a slight perturbation will cause the flow to evolve to a nonsymmetric stable state.

For this calculation the initial shock waves were located at 77% of the chord from the leading edge and the local Mach number just ahead of the shock was $M = 1.172$. During the course of the airfoil motion, the shock waves did not approach closer to the trailing edge than 92% chord location and the maximum local Mach number did not exceed 1.184. Therefore, the flow was consistent with the approximations of the unsteady transonic, small-disturbance theory used in LTRAN2. Also, the shock waves did not reach sufficiently close to the trailing edge that possible interference with the imposition of the Kutta condition might explain the anomalous flow.

Multiple solutions have also been found for the transonic *full potential steady* flow equation by Steinhoff and Jameson.[5] Steinhoff and I will be investigating these anomalous computed flows further with the use of more complete governing equations. The motivation behind this effort is to determine whether the shock wave instability in these calculations also occurs in the Euler equations. If such an instability is found, it could provide an inviscid mechanism for invoking the buffet phenomenon,[6] which also occurs over a narrow range of Mach numbers in the transonic regime.

Acknowledgments

This work was supported, in part, by AFOSR Grant 81-0213A. Dr. Anthony Amos is the Technical Monitor. The third author thanks George Kimberly, Summer Fellowship, NASA Ames Student, Space Biology Research Program, for performing the additional calculations with LTRAN2.

References

[1] Kerlick, G. D. and Nixon, D., "Mean Values of Unsteady Oscillations in Transonic Flow Calculations," *AIAA Journal*, Vol. 19, Nov. 1981, pp. 1496-1498.

[2] Dowell, E. H., Bland, S.R., and Williams, M. H., "Linear/Nonlinear Behavior in Unsteady Transonic Aerodynamics," *AIAA Journal*, Vol. 21, Jan. 1983, pp. 38-46.

[3] Goorjian, P. M., NASA Ames Research Center, private communication, May 1982.

[4] Ballhaus, W. F. and Goorjian, P. M., "Implicit Finite-Difference Computations of Unsteady Transonic Flows about Airfoils," *AIAA Journal*, Vol. 15, Dec. 1977, pp. 1728-1735.

[5] Steinhoff, J. and Jameson, A., "Multiple Solutions of the Transonic Potential Flow Equations," AIAA Paper 81-1019, June 1981.

[6] McDevitt, J., "Supercritical Flow About a Thick Circular-Arc Airfoil," NASA TM 78549, Jan. 1979.

Visualization of Flow Patterns Induced by an Impinging Jet Issuing from a Circular Planform

K. R. Saripalli*

McDonnell Douglas Corporation, St. Louis, Missouri

Introduction

THE development of high-performance V/STOL aircraft powered by lift jets requires knowledge of the complex

Received Aug. 30, 1982; revision received March 22, 1983.

*Research Scientist, McDonnell Douglas Research Laboratories. Member AIAA.